Geometric Methods in Statistics INFT/CSI 877

Problem Set #2 Professor Edward J. Wegman

1. Consider a multivariate normal probability distribution with zero mean and covariance matrix, Σ. Show that the *iso-density contours* (technically called *isopleths*) are hyperellipsoids. If Σ is the identity matrix, the isopleths will be hyperspheres. Calculate the radius of the hypersphere centered at the mean containing 50% of the probability. How is this radius dependent on the dimension? If one calculated the *minimum volume ellipsoid* containing 50% of the observations for a random sample, do you suppose this would be a good estimator of the location and convariance? Why or why not?

2. Compute the length of the arc $\boldsymbol{x} = e^t\ cos(t)\ \boldsymbol{e}_1 + e^t\ sin(t)\ \boldsymbol{e}_2 + e^t\boldsymbol{e}_3$, for $0 \leq t \leq \pi$. Show that $\boldsymbol{x} = t\boldsymbol{e}_1 + sin(t)\boldsymbol{e}_2 + e^t\boldsymbol{e}_3$ for $-\infty < t < \infty$ and $\boldsymbol{x} = log(t)\boldsymbol{e}_1 + sin(log(t))\boldsymbol{e}_2 + t\boldsymbol{e}_3$ for $0 < t < \infty$ are representations of the same oriented curve. Sketch the arc and the curve.

3. Find the curvature and the torsion along the curve

$$\boldsymbol{x} = (t - sin(t))\boldsymbol{e}_1 + (t - cos(t))\boldsymbol{e}_2 + t\boldsymbol{e}_3.$$

Show that the curve

$$\boldsymbol{x} = t\boldsymbol{e}_1 + \frac{1+t}{t}\boldsymbol{e}_2 + \frac{1-t^2}{t}\boldsymbol{e}_3$$

lies in a plane.

4. Find the intrinsic equations for the curve

$$\boldsymbol{x} = e^t(acos(t)\boldsymbol{e}_1 + asin(t)\boldsymbol{e}_2 + b\boldsymbol{e}_3).$$

Find the equation for the normal line.