Geometric Methods in Statistics INFT/CSI 877

Problem Set #1 Professor Edward J. Wegman

1. a. Show $|\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}|\sin(\theta)$.
   b. Show $\boldsymbol{a} \times \boldsymbol{b} \perp \boldsymbol{a}$ and $\boldsymbol{a} \times \boldsymbol{b} \perp \boldsymbol{b}$.
   c. Suppose $\boldsymbol{a} \perp \boldsymbol{b}$. If $\boldsymbol{a} \times \boldsymbol{b} \neq \mathbf{0}$, $(\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{a} \times \boldsymbol{b})$ isaright-handedlinearly independent triplet. Note that $\boldsymbol{a} \times \boldsymbol{b} = \mathbf{0}$ if and only if $\boldsymbol{a}$ and $\boldsymbol{b}$ are linearly dependent.

2. Show by mathematical induction that $\cos(\theta_d) = \frac{1}{\sqrt{d}}$ in general in Example 1.2.

3. Prove **Proposition 2.1:** The orthogonal projection of a point $\boldsymbol{x} = (x_1, x_2, \ldots, x_d)$ onto a line, $\mathcal{L}$: $\frac{y_1}{l_1} = \frac{y_2}{l_2} = \cdots = \frac{y_d}{l_d}$ is a distance $\sum_{i=1}^{d} l_i x_i$ from the origin $\mathbf{0}$.

4. Prove **Proposition 2.2** If a second line has direction cosines l_i', then ϕ is the angle between the two lines where $\cos(\phi) = \sum_{i=1}^{d} l_i l_i'$. The lines are orthogonal if

   $\sum_{i=1}^{d} l_i l_i' = 0.$

5. Consider the following 4-dimensional points:
   (1, 1.4, 6, -2)
   (1, 8.3, 4, 3)
   (-2, 4, 3.1, 12)
   (3.1, 2.1, 1.1, .1)
   (-2, -2, -2, -2).

Calculate the content of the 4-dimensional simplex generated by these points. Is the origin, (0,0,0,0) interior to this simplex?