SUBSAMPLING – BASED INFERENCE FOR PARAMETERS OF THE ATMOSPHERIC BOUNDARY LAYER

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Abstract

Employing the computer-intensive subsampling methodology can considerably improve the statistical validity of atmospheric data analysis. In particular, it makes possible statistically significant comparisons between statistical characteristics of the atmospheric boundary layer computed from observational and large-eddy simulation (LES) data sets. This work illustrates these possibilities by examining the vertical velocity variance estimates from Project LESS (Lake-Effect Snow Studies) aircraft measurements and from LES of the Project LESS event, where subsampling also helps to explain some discrepancies between observations and LES results.

1. Introduction

Our understanding of atmospheric turbulence is based on observations and, increasingly, on modeling. Field studies provide data in the form of time series (the behavior of a meteorological element is recorded along one spatial or temporal variable); modeling supplies three-dimensional data; and statistical characteristics are computed from these data by time or space averaging. Such approximations of ensemble averages are valid for *ergodic* random functions. Ergodic theorems (e.g., Tempelman 1992) state conditions for *stationary* random processes or *homogeneous* random fields to be ergodic. However, time series in practice are typically not stationary since meteorological fields may exhibit varied behavior over different regions, although horizontal fields in large eddy simulation (LES) and direct numerical simulation (DNS) can often be considered homogeneous.

In practice, ergodicity is difficult to test and usually accepted for the stationary processes under study based on physical arguments (see discussion in Yaglom 1987). To deal with nonstationarity, Gluhovsky and Agee (1994) suggested a method that permits a selection of intervals within the record, where the time series can be considered stationary (see also Witt et al. 1998). However, even if data are stationary or homogeneous, record lengths from observations (typically aircraft flight legs) are usually inadequate for definitive statistical estimation. In contrast to other areas of turbulence research where the time and space scales of turbulence are much smaller, in the atmosphere, length averages can be statistically unreliable.

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For example, in this paper we examine data collected under Project LESS (Lake-Effect Snow Studies) in the winter of 1983-84 over Lake Michigan. For these data, accurate estimates of higher moments may require hundreds, even thousands of kilometers of stationary records (Gluhovsky and Agee 1994). Lake Michigan is not that large.

In atmospheric *modeling*, the averaging problem is less acute since, as was suggested by Wyngaard (1983), averaging over an area provides considerable advantages over averaging over a line. Indeed, suppose that a record of length T is available of a stationary process $W(t)$ (the vertical velocity, for example, where *t* is the position on a line as in aircraft measurements) with mean μ and covariance function $B(\tau)$. Let the parameter of interest be mean μ, estimated with the sample mean $m_T = T^{-1}\int_0^T W(t)dt$. If the covariance function satisfies certain regularity conditions, then the sample mean m_T is asymptotically normal, with mean μ and variance $\sigma_{m_T}^2 = T^{-1}\int_{-\infty}^{\infty} B(\tau)d\tau$ (e.g., Yaglom 1987). Statistical characteristics of an *isotropic* random field may be estimated from measurements both along any segment of length T of a straight line and over an area (say, a square of side $\tilde{T}$). Then the two estimators of the same sample mean, m_T and $m_{\tilde{T}^2} = \tilde{T}^{-2}\int_0^{\tilde{T}}\int_0^{\tilde{T}} W(t_1,t_2)dt_1dt_2$, with the respective variances, $\sigma_{m_T}^2$ and $\sigma_{m_{\tilde{T}^2}}^2 = \tilde{T}^{-2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B(\tau_1,\tau_2)d\tau_1 d\tau_2$, enable us to determine the length of the side of the square that provides the same accuracy ($\sigma_{m_T}^2 = \sigma_{m_{\tilde{T}^2}}^2$):

$$\tilde{T} = \sqrt{\left(\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} B(\tau_1,\tau_2)d\tau_1 d\tau_2 \Big/ \int_{-\infty}^{\infty} B(\tau)d\tau\right) T} \; .$$

For example, for a field with model covariance function $B(\tau) = e^{-\alpha|\tau|}$,

$$\tilde{T} = \sqrt{(\pi/\alpha)\, T} \; .$$

The problem of averaging has always been of great concern for atmospheric scientists (e.g., Lumley and Panofsky 1964, Wyngaard 1983, Sreenivasan et al. 1978, Lenschow et al. 1994). Many new observational studies combined with the flow of data from LES and DNS have renewed interest in examining the significance of turbulence statistics. Of particular interest are the comparisons of LES and DNS results with observations where significant discrepancies have to be explained.

To address these problems, we have chosen the records of the vertical velocity from aircraft measurements under Project LESS and the LES of the Project LESS event to estimate the vertical velocity variance with the sample variance $V(W)$.

Computations of statistical characteristics should always include estimates of the standard error of the estimator and the determination of confidence intervals, especially in atmospheric studies where having enough observations to supply data for computer models is continually a problem. Politis and Romano (1993, 1994; see also Politis et al. 1999) have developed a general method for constructing confidence intervals by appropriate use of subsampling. The method is based on recomputing a statistic over subsamples of the data, and these recomputed values are used to build up an estimated sampling distribution. The method works under extremely weak conditions, and it applies to independent, identically distributed observations as well as to dependent data situations, such as time series and random fields.

2. Data

The 10 January 1984 outbreak of a polar continental air mass over the Great Lakes region resulted in the development of convective boundary layer due to heating by the warm lake waters. Under Project LESS, two research aircraft flew over Lake Michigan at five levels between the west and east shorelines resulting in 5 vertically stacked levels of boundary layer data. Figure 1 shows the records of the vertical velocity at these five levels taken at $70\ \mathrm{ms}^{-1}$ flight speed and 20 Hz sampling rate. Gluhovsky and Agee (1994) subjected these data to their test for stationarity and computed (from stationary data) statistical characteristics of the vertical velocity and their confidence intervals. The results indicated that one could extract three regions over Lake Michigan (shown by heavy solid lines in Figure 1) where statistical properties of vertical velocity exhibit statistically significant differences. The existence of these regions reflects changing physical conditions in the convective boundary layer. The test for stationarity has also revealed that there were no segments of stationary data longer than 35 km.

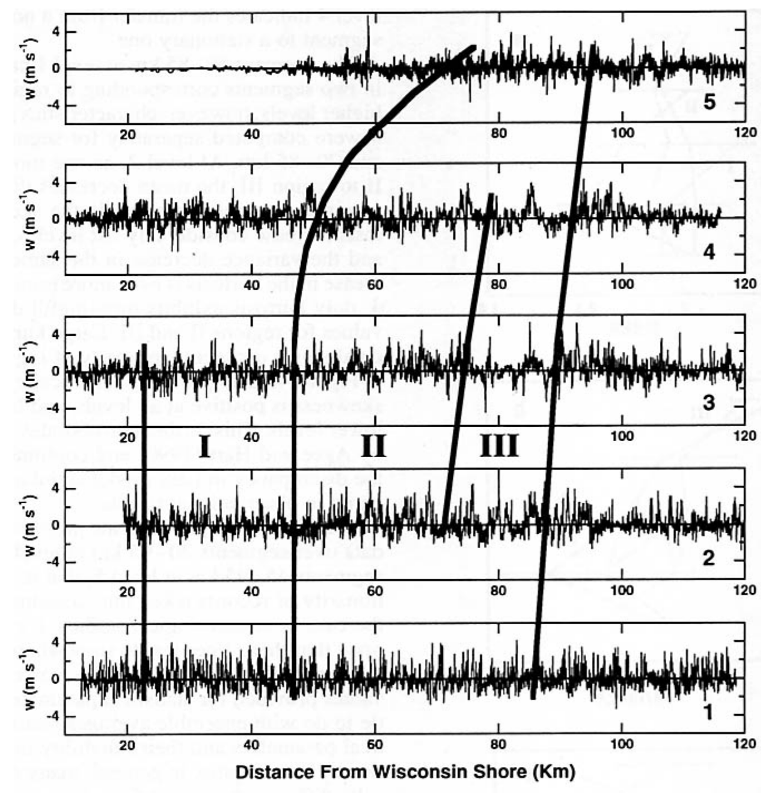

Fig. 1. Plots of 20-Hz aircraft vertical velocity measurements for flight levels 1 through 5 (from Gluhovsky and Agee 1994).

The observational data sets we employed consisted of four vertically stacked *stationary* records of the vertical velocity (from four lower levels), each containing about $N = 8500$ observations collected every 3.5 m.

The simulated data sets were obtained by running the latest version of the Sorbjan (2001) LES model, tuned to model the Project LESS event. It used $64\times64\times40$ grid points covering a domain of about $5\times5\times2$ km. Four appropriate horizontal samples of $64^2 = 4096$ data points each were chosen for comparisons with the field data.

3. Results

3.1 Discrepancies between observations and modeling

Figure 2 presents two profiles for the sample variance of the vertical velocity at levels 1 – 4. One profile (shown by solid line *I*) was computed from Project LESS data; the other (shown by dashed line *II*) was obtained from LES.

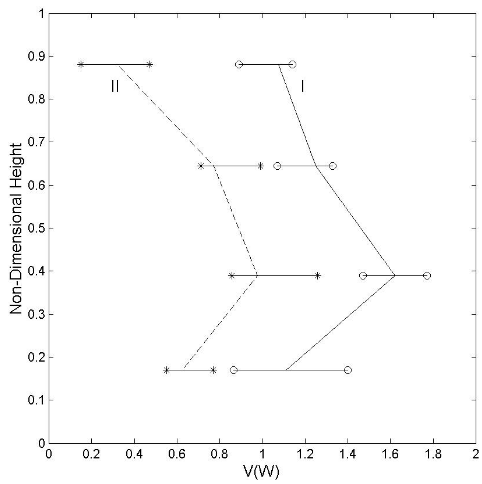

Fig. 2. Vertical velocity variance profiles from LESS field data (solid line I) and from LES model data (dashed line II). Circles and stars denote respective 90% confidence limits.

90% confidence intervals in Figure 2 indicate statistically significant differences between the two kinds of data. Limited domains in current LES models (e.g., $5\times5\times2$ km) eliminate the effects of larger wavelength structures present in the atmosphere. LES models are also limited by a finite grid size. Thus, such models are intrinsically band-pass filters, and to ensure that a comparison of "oranges with oranges" is made, a band-pass filtering should be performed on the field data.

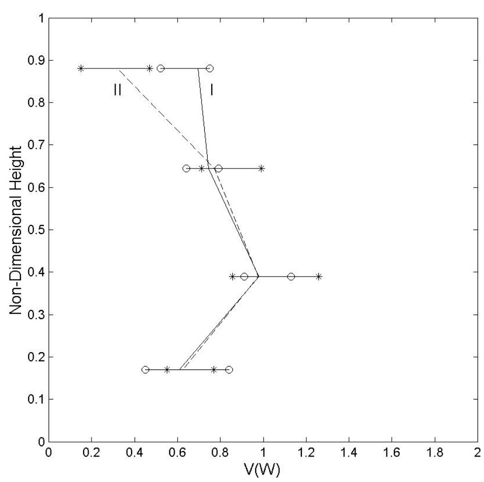

Fig. 3. Vertical velocity variance profiles from *filtered* LESS field data (solid line I) and from LES model data (dashed line II). Circles and stars denote respective 90% confidence limits.

Figure 3 demonstrates the effect of the filtering – for lower levels, there is no need to invoke physical arguments to account for the discrepancy between vertical velocity variance from observations and from LES.

3.2 Confidence Intervals

Confidence intervals shown in Figures 2 and 3 were determined by using the subsampling methodology based on the values of $V(W)$ computed over subsamples, or blocks, that retain the dependence structure of the observations. In the case of the field data, for every level, a one-dimensional array of about $N = 8500$ observations produced $N - b + 1$ such blocks, each containing b data points $\{W_i,\ W_{i+1}, \ldots,\ W_{i+b-1}\}$. For the modeled data, the arrays were two-dimensional, each contained $N = n^2 = 64^2 = 4096$ data points and produced $(n - a + 1)^2$ blocks in the form of a square with side a, i.e. containing $b = a^2$ observations.

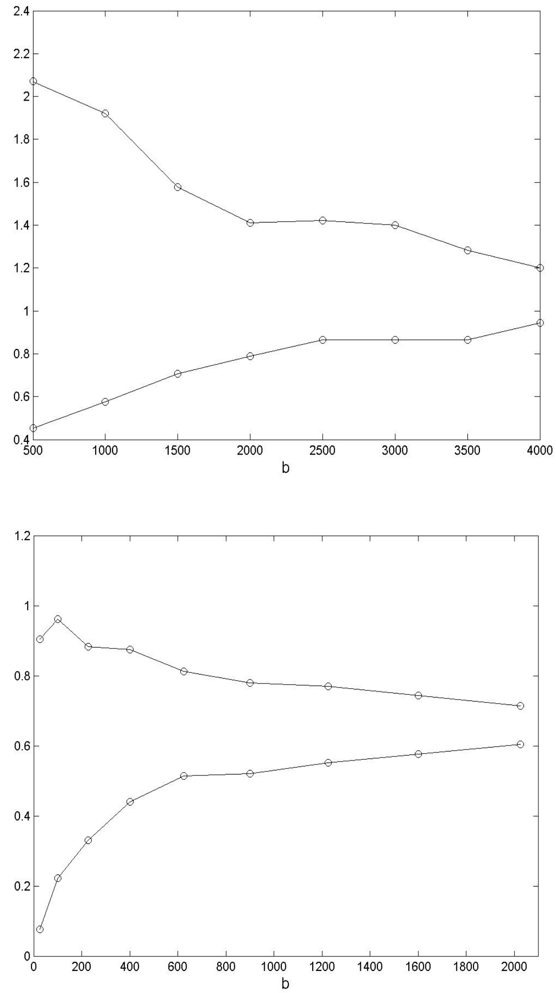

Fig. 4 Upper and lower 90% confidence limits of $V(W)$ as functions of block size b from observational (first plot) and modeled (second plot) data.

The choice of the block size b presents the most difficult practical problem in using the subsampling method shared by all blocking methods. In this study, b was selected following the minimum volatility method (Politis et al. 1999) that recommends values of b from a range where confidence intervals are relatively stable.

Figure 4 shows the typical dependence of confidence intervals on the block size b for the field and the modeled data. Ranges of relative stability of confidence intervals employed in this study for determining b were, respectively, $2000 < b < 3000$ and $600 < b < 1200$.

4. Conclusions

In this study, subsampling methodology was employed to compute confidence intervals, thus ensuring the statistical validity in a practical framework of atmospheric measurements and modeling where standard assumptions – such as Gaussianity, linearity, or even homogeneity – are not guaranteed to hold. For the specific problem of the discrepancy between LES representation of the vertical velocity variance and observations, it allowed us to clearly demonstrate that in some cases there is no need to bring up physical arguments to account for this discrepancy. Due to inadequate domain size in current LES models and a finite grid size, larger scales and very small scales are underrepresented in the frequency spectrum in contrast to field observations, where they may contribute considerably to the formation of the vertical velocity variance profile. Thus, band-pass filtering is necessary to make appropriate comparisons between statistical characteristics computed from observational field data and LES model data sets.

It was also demonstrated that box area measurements and modeling offer considerable advantages over linear flight data. Confidence intervals computed from a 5×5 km domain proved comparable in size with those from 30km flight legs (see also the model example in the Introduction). An increase in accuracy, particularly for higher moments, would require considerably longer flight legs where the field is unlikely to remain homogeneous and to preserve its local physical properties that are often of prime interest. Therefore, an alternative way of data collection, box area measurements (lidars, remote sensing), is needed.

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