Bayesian Assessment of Uncertainty in Deterministic Environmental Exposure Models

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Motivation

- Inference in an exposure assessment framework.
- Make full use of sparse and expensive data.
- Use information from other sources such as expert opinion.

Bayesian Approach

- Deterministic model M maps inputs θ to outputs ϕ.
- $M(\theta)$ are the “induced” outputs.
- $p(\theta)$ is a prior on θ.
- $p(\theta)$ may be determined from expert opinion.
- $L_{inp}(\theta)$, $L_{out}(\phi)$ are likelihoods on θ and ϕ.

- $\pi^{[\theta]}(\theta)$ is the posterior distribution of θ

$$\pi^{[\theta]}(\theta) \; \propto \; p(\theta)\, L_{inp}(\theta)\, L_{out}(M(\theta))$$

- $\pi^{[\phi]}(\phi)$ is the distribution of $\phi = M(\theta)$ when $\theta \sim \pi^{[\theta]}(\theta)$.

Sampling Importance Resampling

To evaluate $\pi^{[\theta]}(\theta)$ we use the SIR algorithm of Rubin (1988):

1. Draw k values of θ from the prior distribution of θ, $p(\theta)$.

2. Find $\phi_i = M(\theta_i)$ for $i = 1, \ldots, k$.

3. Find weights $w_i = L_{inp}(\theta_i) L_{out}(\phi_i)$, $i = 1, \ldots, k$.

4. Draw m values of θ_i from the discrete distribution of k values of θ_i's with probabilities w_i. This represents an approximate sample from $\pi^{[\theta]}(\theta)$.

Note: A Monte Carlo approach performs steps 1 & 2.

Poly-Chlorinated Biphenyls

- Old fluorescent lighting fixtures, televisions and fridges may contain PCBs.
- Old transformers and capacitors contain PCBs.
- Hazardous waste containing PCBs has been disposed of at sea and on land, and PCBs can be released to water, marine life, land and air.

New Bedford Harbor

- NBH is the site of a Superfund cleanup.
- Between World War II and the 1970s local industries disposed of waste containing PCBs into the harbor.
- Dredging to remove the waste occurs while PCB contaminated sediment is exposed to the air at low tide. PCBs can be transported from the sediment to air or water and then travel to remote human receptors, and transport is increased during clean-up.

Data

- Researchers have measured air, soil, house dust, tap water and tree, leafy and root plant PCB concentrations at many locations throughout the NBH area.
- 59 individual PCB congeners were targeted for attention. We use the sum of those congeners in this analysis.

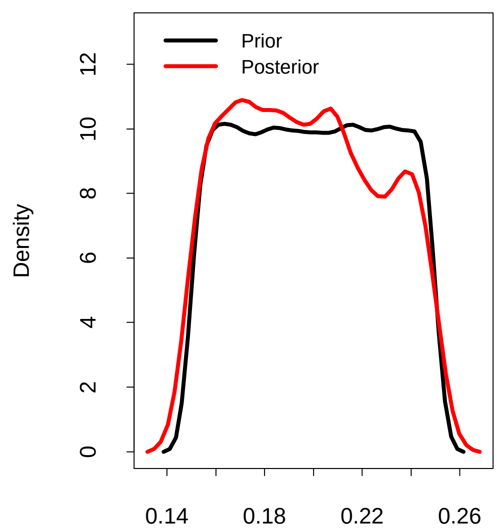
Soil Model

A model for predicting the concentration of PCB in soil from the concentration of PCB in air:

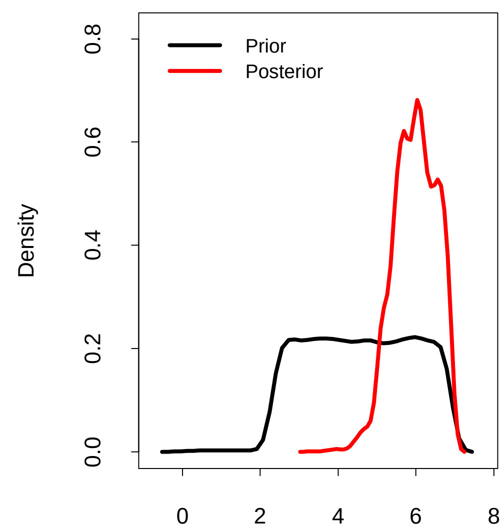
$$Cs = \frac{Ca\ Vd}{b\,\rho\,D} \quad mg/g$$

Input	Explanation	Units
Vd	deposition velocity	m/d
b	decay constant in soil	$1/d$
ρ	density of the soil	g/m^3
D	mixing depth of soil	m
Ca	PCB concentration in air	mg/m^3

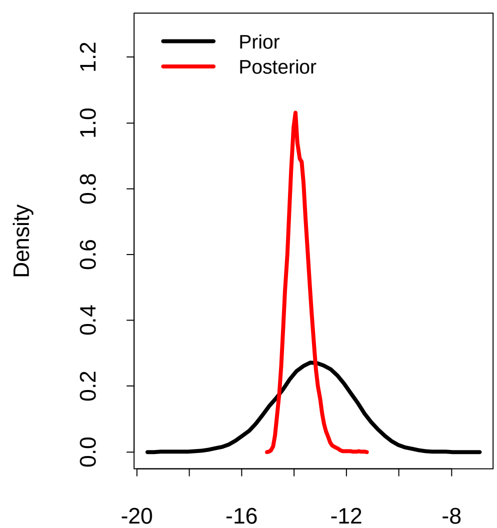
Posterior Distributions for Inputs



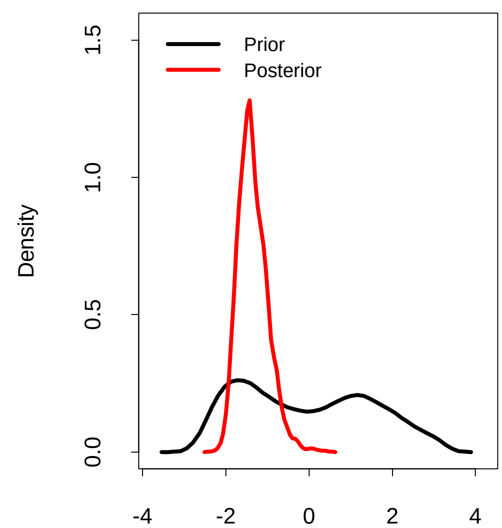
Mixing Depth -- D



Ln(Deposition Velocity) -- Ln(Vd)

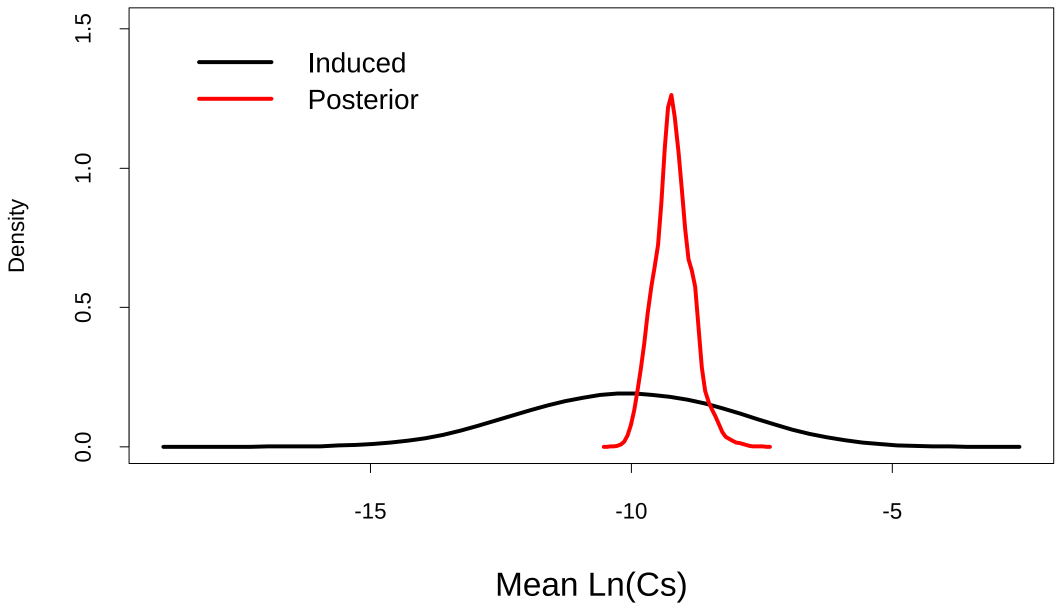


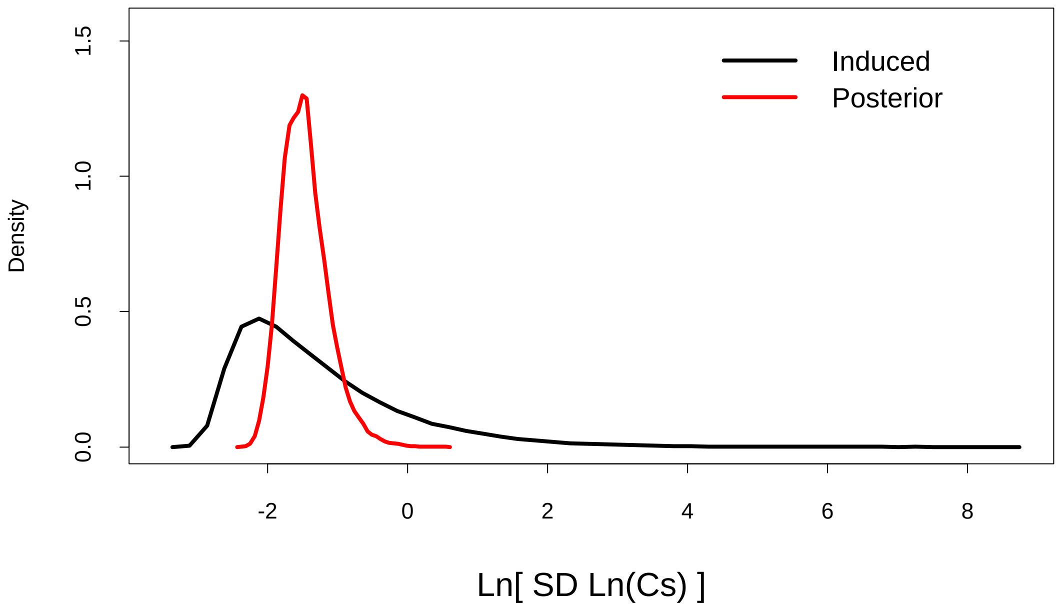
Mean Ln(Ca)



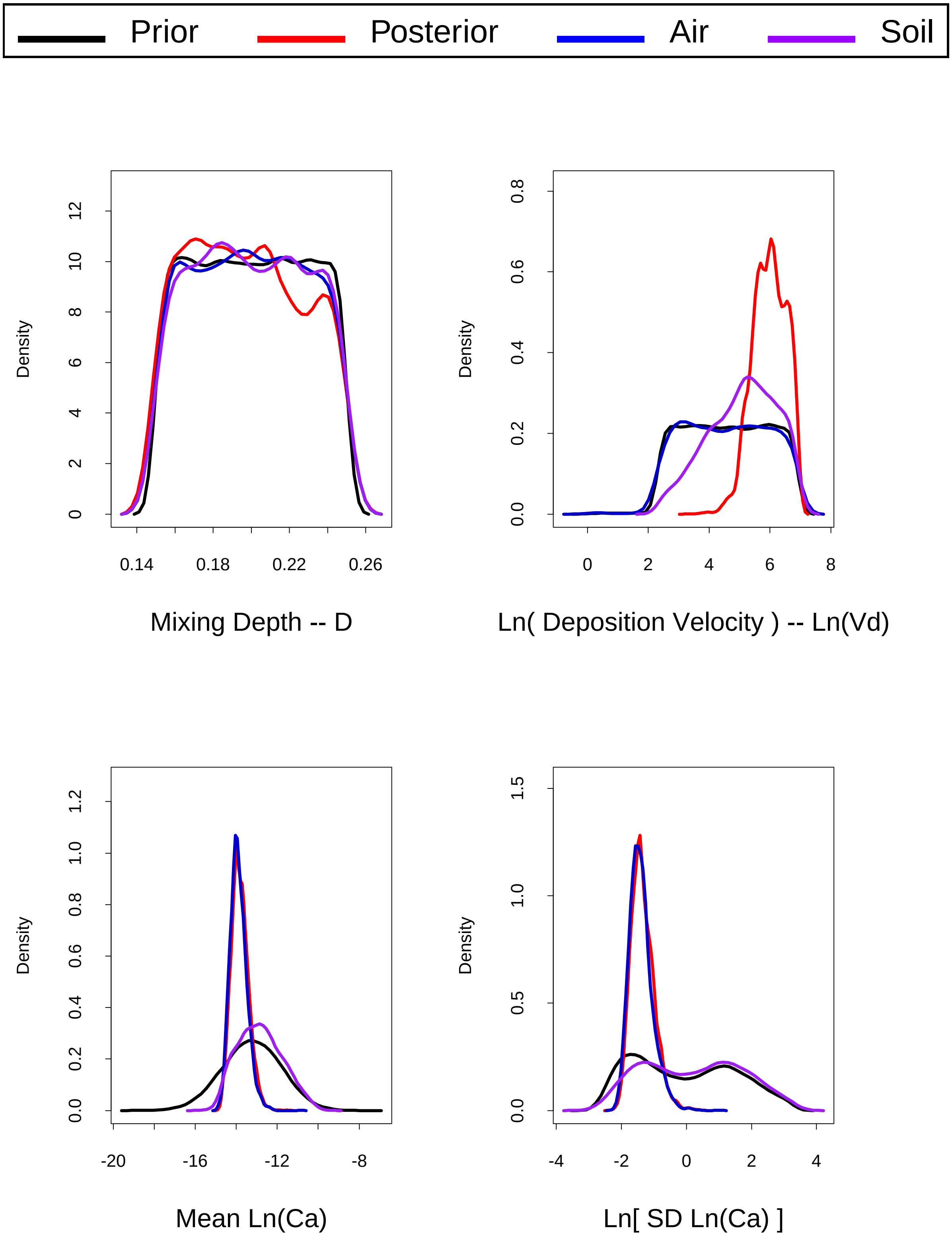
Ln[SD Ln(Ca)]

Posterior Distributions for Outputs

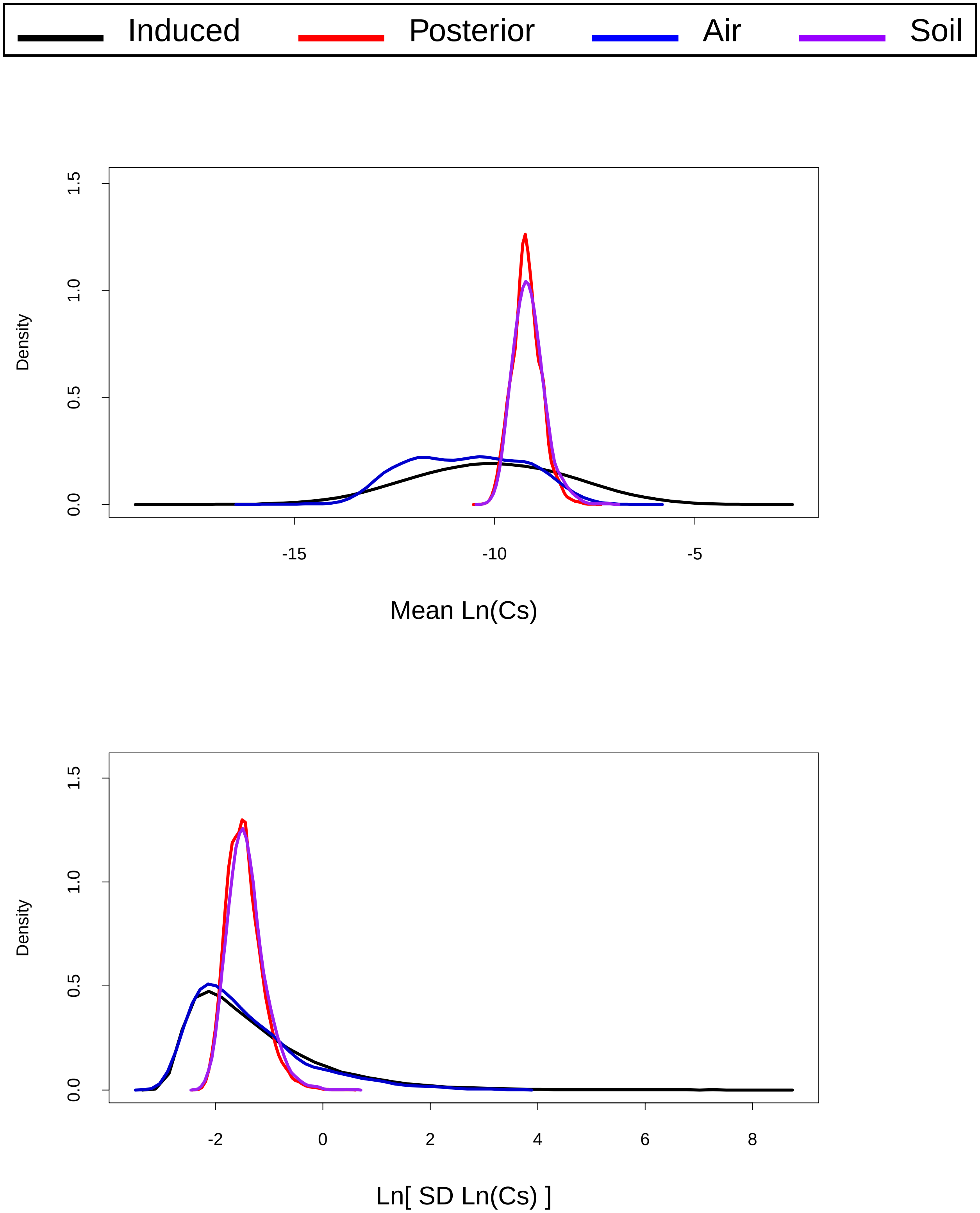




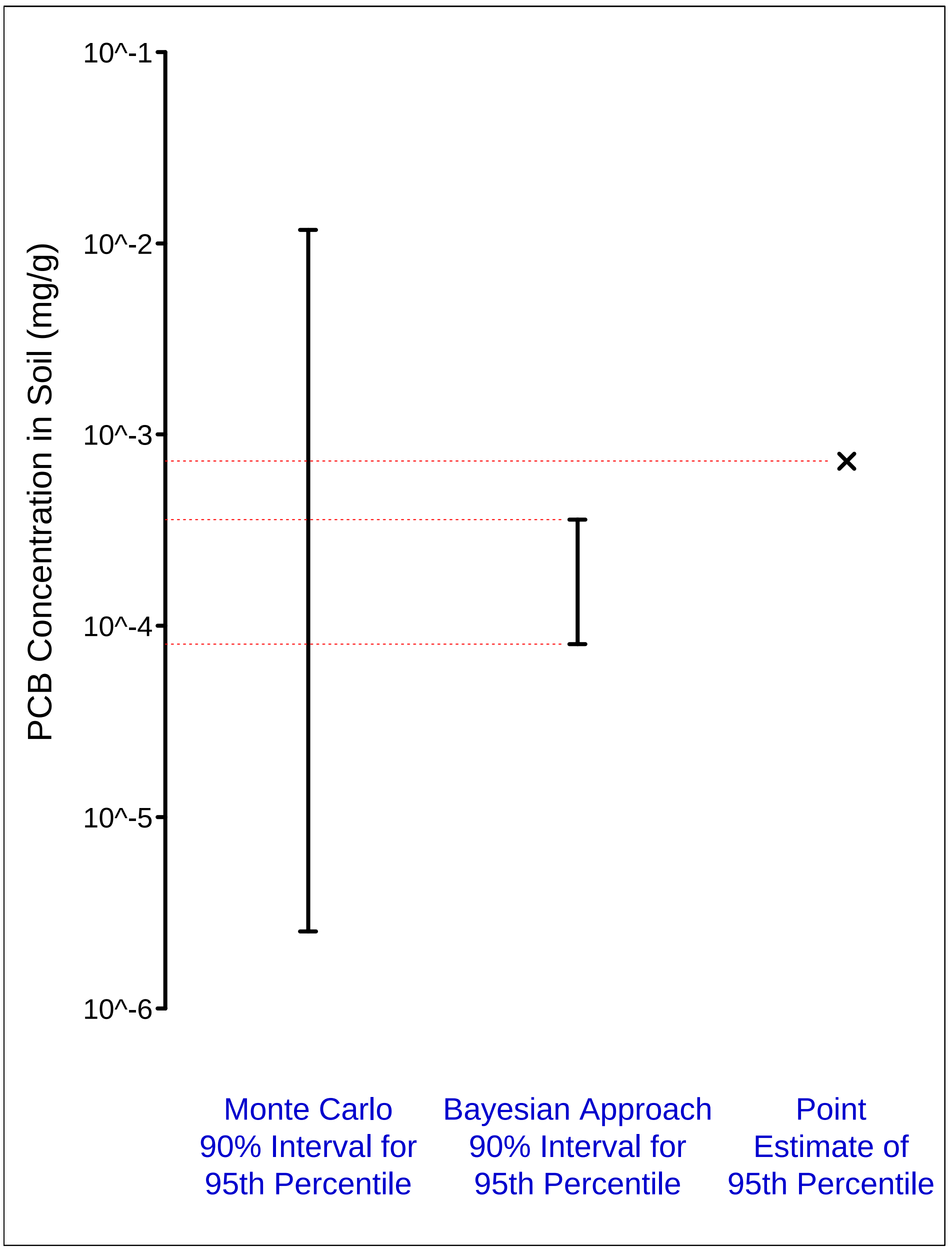
Posterior Distributions for Inputs



Posterior Distributions for Outputs



Comparison of Methods



Conclusions

- Application of standard Bayesian methodology to deterministic models when the analytic form of the posterior distribution is unknown.
- This approach takes into account all information (both prior and data based) on the inputs and outputs to a model. Data can be combined with expert knowledge.
- The approach then allows the data to update the inputs and outputs.